Parikh Automata and Friends

On the Expressiveness of Parikh Automata and Related Models (NCMA11)

Bounded Parikh Automata (WORDS11)

M. Cadilhac[1], A. Finkel[2], and P. McKenzie[1]

1: Université de Montréal

2: ENS CACHAN

Tübingen, September 22th, 2011

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- In model checking, properties described by automata

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- Small complexity classes (within NC^2)

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Outline

Parikh Automata

Affine Parikh Automata

Letter Parikh Automata

Bounded languages of PA

Corollaries and Further Work

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Parikh automata

Preliminary: semilinear sets

Definitions

- *Linear* set: of the form $E = \{\vec{c}_0 + \sum_{i=1}^{m} \vec{c}_i \cdot k_i \mid k_i \in \mathbb{N}\}$

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Why are they natural? One of many reasons:

Theorem ([Parikh, 1966])

With $\Sigma = \{a_1, \ldots, a_n\}$ *and* $w \in \Sigma^*$, *let*
$\text{Parikh}(w) = (|w|_{a_1}, \ldots, |w|_{a_n}) \in \mathbb{N}^n$ *be the Parikh image of* w.

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L *context-free* $\Rightarrow$ $\text{Parikh}(L)$ *semilinear*

Start with an example

Parikh automaton: semilinear constraint on Parikh(a path)

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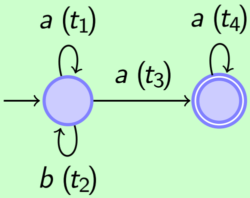

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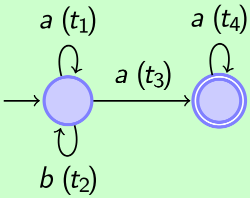

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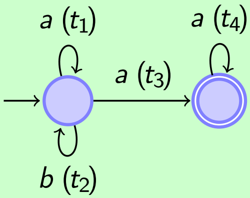

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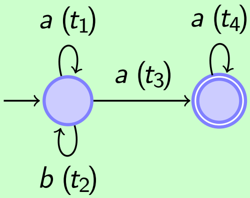

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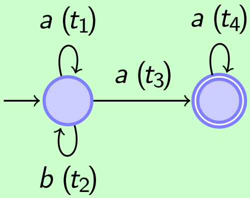

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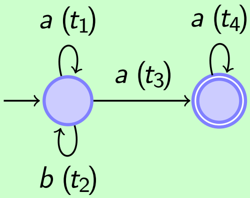

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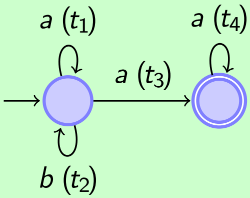

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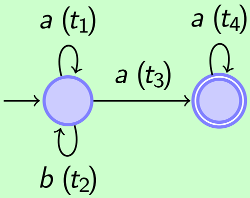

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Parikh(traced run):
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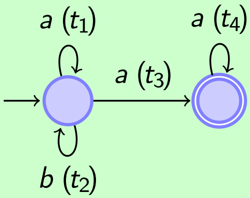

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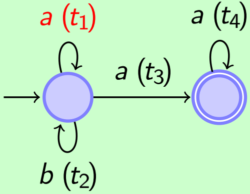

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Parikh(traced run): (1, 0, 0, 0)

$\pi = t_1$

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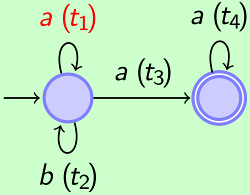

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Parikh(traced run): (2, 0, 0, 0)

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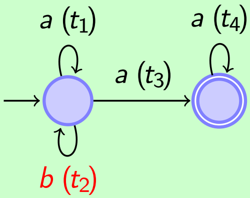

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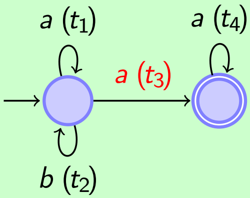

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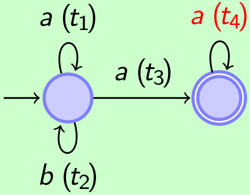

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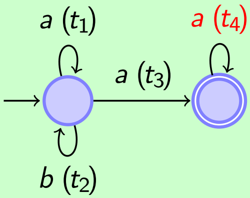

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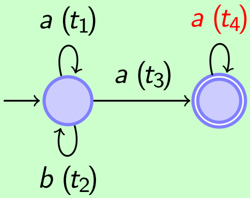

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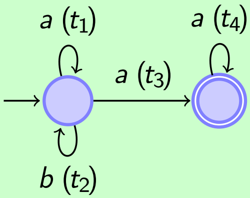

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Parikh(traced run): $(2, 1, 1, 3) \in C$

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Parikh automata

The formal definition

Definition ([Klaedtke and Rueß, 2003])

- *Parikh automaton* (PA): a pair (A, C) with:
  - A a finite automaton of transition set δ
  - $C \subseteq \mathbb{N}^{|\delta|}$ semilinear

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Why is PA a relevant model?

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- Equivalent to extended automata over $(\mathbb{Z}^k, +, 0)$ of [Mitrana and Stiebe, 2001]

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Parikh automata

Expressiveness: main (simple) tool

Lemma

For $L \in$ *PA* , $\exists p, l \in \mathbb{N}^+$ *s.t. all* $w \in L \cap \Sigma^{>l}$ *can be written* *uvxvz with:*

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For $L \in$ PA, $\exists p, l \in \mathbb{N}^+$ s.t. all $w \in L \cap \Sigma^{>l}$ can be written uvxvz with:

- $0 < |v| \geq p$
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- $|uvxv| < l$
- $uv^2xz, uxv^2z \in L$

- Ex.: COPY $\notin$ PA $(a^p b)^l \#(a^p b)^l$

Parikh automata

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Parikh automata

Decidability and closure properties

- Decidability results:

	$= \varnothing$	$= \Sigma^*$	is finite	$\subseteq$	is regular
DetPA	D	D	**D**	**D**	?
PA	D	U	**D**	**U**	**U**

Parikh automata

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- Closure properties:

	$\cup$	$\cap$	$\cdot$	$\rightarrow$	h	h^{-1}	c	$*$
DetPA	Y	Y	**N**	Y	**N**	Y	**Y**	**N**
PA	Y	Y	Y	N	Y	Y	**Y**	N

Parikh automata

PA and RBCM

Definitions

- Counter machine: two-way automaton with 0-test counters

Parikh automata

PA and RBCM

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- $PA = RBCM$
- $DetPA \subsetneq DetRBCM$
- $NSUM = \{a^n \spadesuit b^{m_1} \# \cdots \# b^{m_k} \clubsuit c^{m_1 + \cdots + m_n}\} \in DetRBCM \setminus DetPA$

Outline

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Affine Parikh Automata

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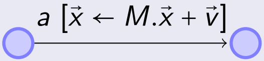
Affine Parikh automata

A quick definition

Definition

- *Affine Parikh automaton* given by:
  - A finite automaton
  - A labelling of the transitions by affine functions (on $\mathbb{Q}$ or $\mathbb{N}$)
  - A semilinear set (for $\mathbb{Q}$, FO)



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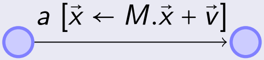

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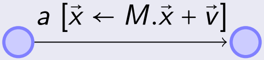

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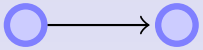

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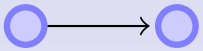

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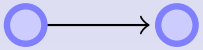

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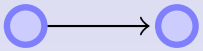

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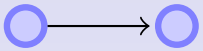

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- Dyck $\notin$ APA
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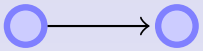

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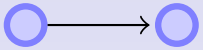
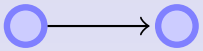


Affine Parikh automata

Decidability and closure properties

- Positive properties: (Det)APA closed under $\cap$, $\cup$, h^{-1}; APA under concat, DetAPA under complement



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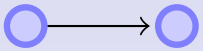

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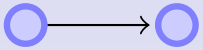

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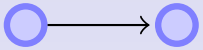

Letter Parikh automata

A simple restriction

Definition

PA without distinguishing transitions with same label

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$$R\restriction_C = R \cap \text{Parikh}^{-1}(C)$$

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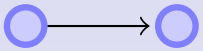

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Leads to:

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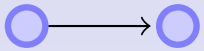
Letter Parikh automata

A link with PA

But PA $= h(\text{LPA})$ helps in proving Parikh-boundedness:



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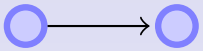

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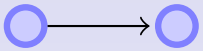

Bounded languages of PA

Definition: bounded languages

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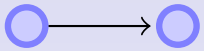

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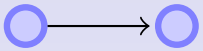

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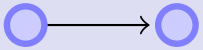

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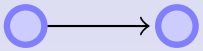

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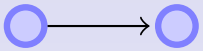

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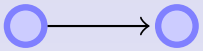

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- ▸▸ BSL intensively studied, e.g., Ginsburg & Spanier, 60's

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LPA

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$L \subseteq w_1^* \cdots w_n^*$

Iter. set

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BSL

Bounded ∧ SL iter. set

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Parikh(*any L in PA*) *semilinear*

PA closed under h^{-1}, $\cap$

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Theorem, restated:

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$\text{PA} \cap \text{BOUNDED} \subsetneq \text{BSL} \subsetneq \text{DetPA} \cap \text{BOUNDED}$

Parikh(*any L in PA*) *semilinear*
PA closed under h^{-1}, $\cap$

Rest of this section

SL set

$\vec{c_0} + \sum_i \vec{c_i} \cdot k_i$

(Det)PA

automaton constraining #transitions

RBCM

Rev. bounded 0-test counters

(Det)APA

PA with func

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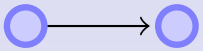


LPA

automaton constraining #letters

Bounded

$L \subseteq w_1^* \cdots w_n^*$

Iter. set

$\{\vec{i} \mid \vec{w}^{\vec{i}} \in L\}$

BSL

Bounded ∧ SL iter. set

Outline

Parikh Automata

Affine Parikh Automata

Letter Parikh Automata

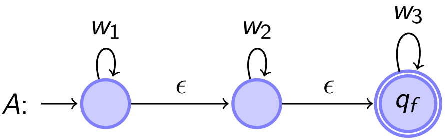
Bounded languages of PA

- BSL ⊆ ℕ-DetAPA with some property X
- ℕ-DetAPA with this property ⊆ DetPA
- Corollaries

Corollaries and Further Work

BSL ⊆ ℕ-DetAPA with some property X

- Let $L \subseteq w_1^* w_2^* w_3^* \in$ BSL. Then a PA describes L:



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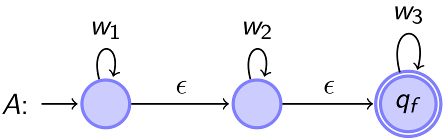

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$$\text{Parikh}(\pi_1) \in C \Leftrightarrow \text{Parikh}(\pi_2) \in C$$

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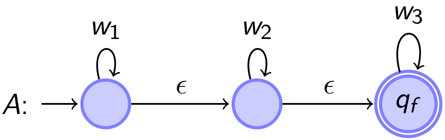

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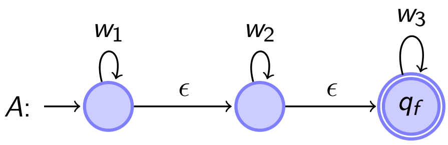

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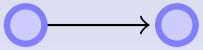

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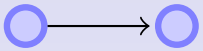

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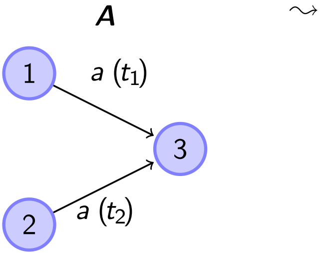

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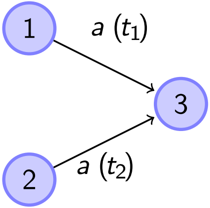

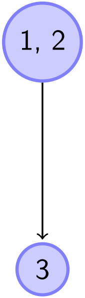

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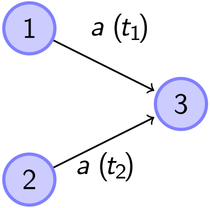

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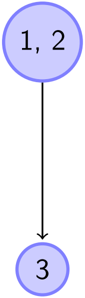

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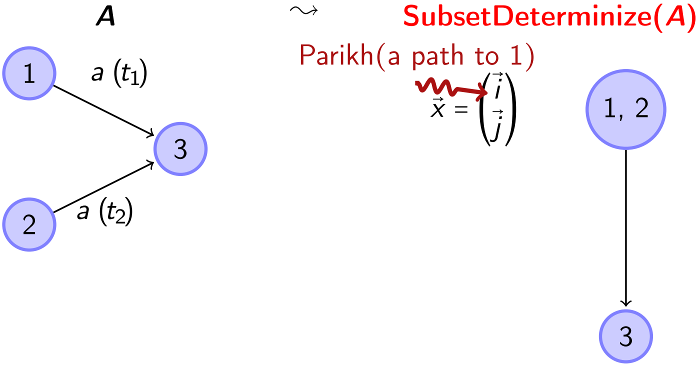

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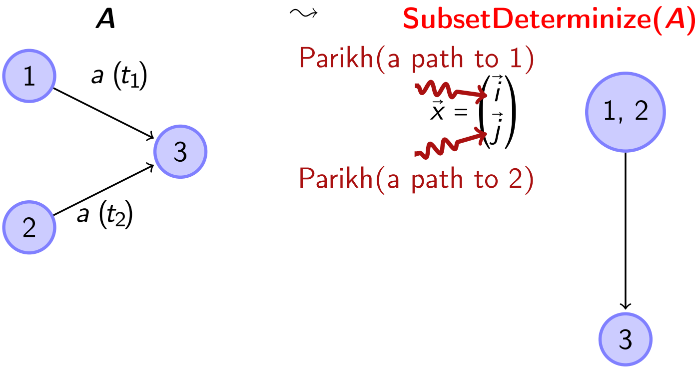

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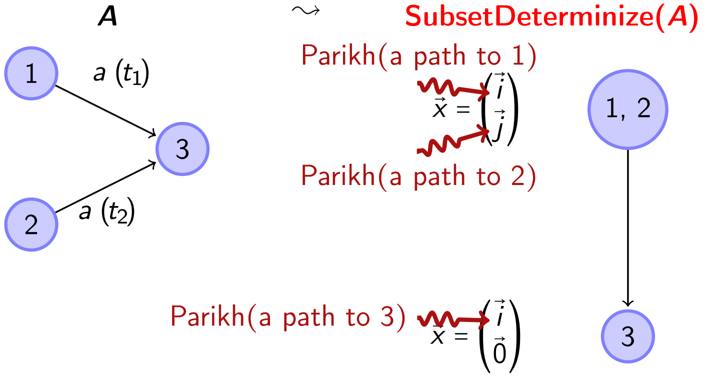

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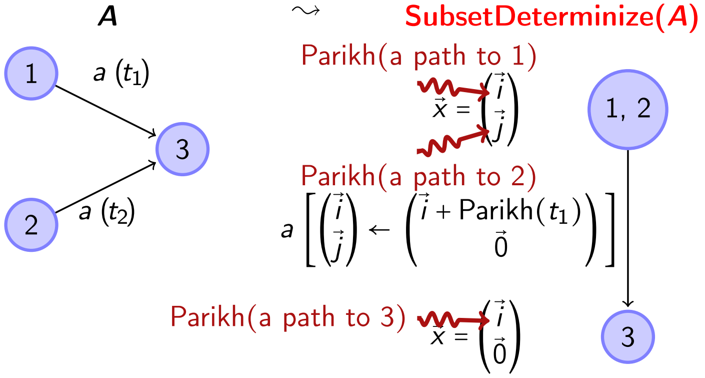

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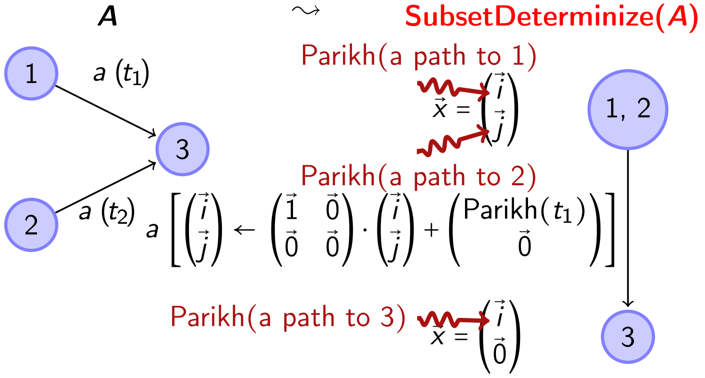

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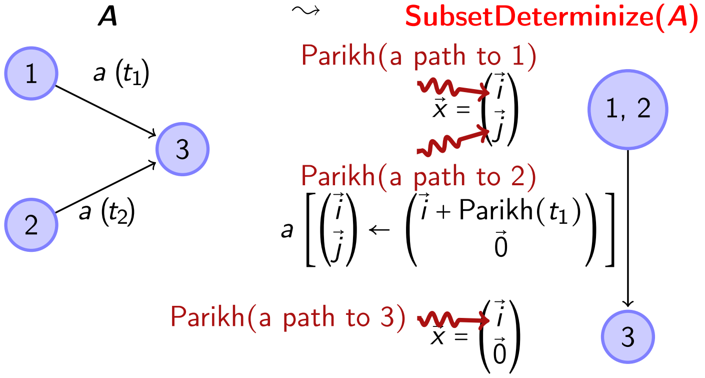

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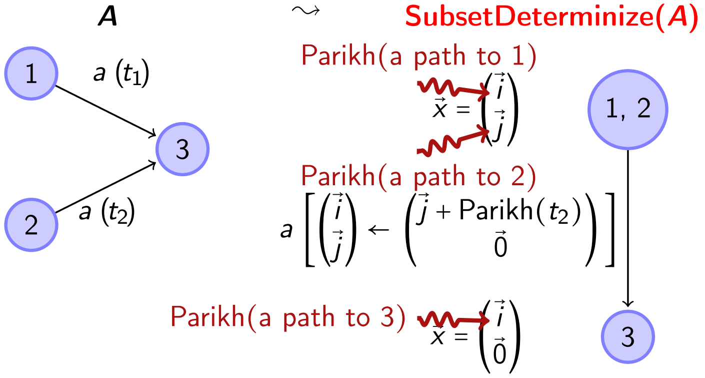

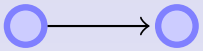

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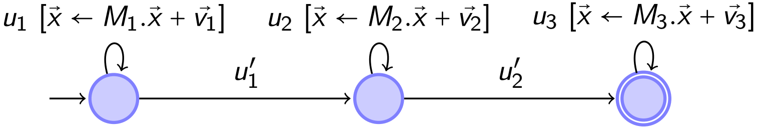
- We can assume the DetAPA, of language L, is of the form:



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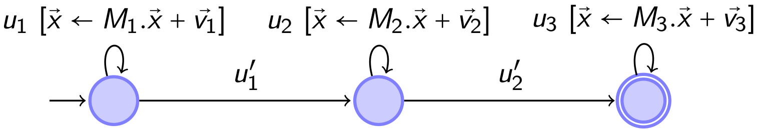

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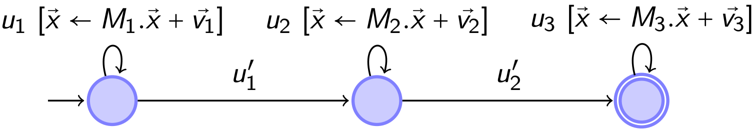

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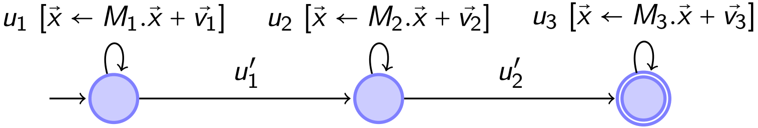

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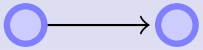

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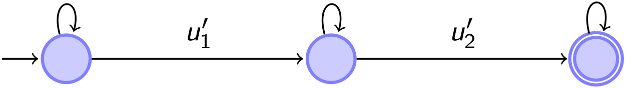

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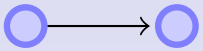

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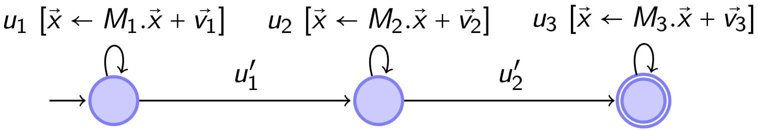

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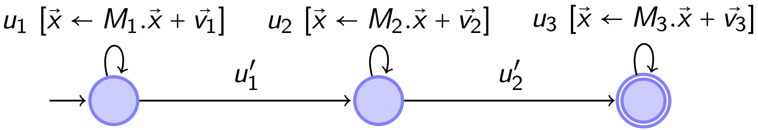

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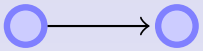

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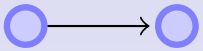

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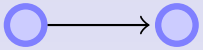

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What implications to algebraic/logic characterizations?

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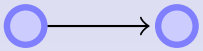

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